APPLICATION OF NON-CRITICAL STRING THEORY TO NON-PERTURBATIVE PHYSICS AND OPEN-CLOSED STRING DUALITY

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Abstract:

The exact FZZT brane partition function for topological gravity with matter is computed using the dual two-matrix model. We show how the effective theory of open strings on a stack of FZZT branes is described by the generalized Kontsevich matrix integral, extending the earlier result for pure topological gravity. Using the well-known relation between the Kontsevich integral and a certain shift in the closed-string background, we conclude that these models exhibit open/closed string duality explicitly. Just as in pure topological gravity, the unphysical sheets of the classical FZZT moduli space are eliminated in the exact answer. Instead, they contribute small, non-perturbative corrections to the exact answer through Stokes’ phenomenon.

Introduction:

Open/closed string duality is one of the most intriguing facets of string theory. Recently, there has been renewed interest in "minimal string theory," i.e. non-critical strings with c < 1. (For a list of references to recent work, see e.g. [1].) These models are dual to certain zero-dimensional gauge theories (i.e. matrix models), providing a tantalizing hint that some form of open/closed string duality is at work here. Because these systems are integrable, it is natural to expect that c < 1 string theories exhibit open/closed string dualities which are explicitly demonstrable. This strongly motivates us to study these models further. It states that dynamical processes involving g open strings can be formulated strictly in terms of closed strings, and vice versa. Often, the open string side of the correspondence can be reduced to an ordinary gauge theory. In such cases, the correspondence can provide a promising way to address outstanding issues of quantum gravity by reformulating them as questions in gauge theory.

Since our goal here is to understand open/closed string duality in minimal string theory, it is natural to consider the dynamics governing the open strings ending on FZZT branes. Exactly this question was addressed for the case of the c = −2 model – known as pure topological gravity from the work of [8] – first from the point of view of string field theory in [9], and later using the double-scaled matrix model in [6]. Both approaches found that the open strings ending on FZZT branes in this background are described by the matrix integral of Kontsevich [10]. By identifying the precise deformation of the closed string background associated with the presence of the brane, the authors of [6,9] showed concretely how Kontsevich’s formulation of two dimensional gravities can be thought of as a kind of open/closed string duality. This paper is organized as follows. We begin in section 2 by
formulating the \((p, 1)\) minimal string theories as a double-scaling limit of the two matrix model. We compute the partition function of a stack of FZZT branes in this background and obtain the generalized (matrix) Airy function. We also explore the Stokes and anti-Stokes lines for the FZZT partition function. In section 3, we use the correspondence between macroscopic loop operators and the local closed string operators to formulate the open/closed string duality for the \((p, 1)\) model. Finally, we conclude in section 4 with a discussion of open problems and relation to other work. The appendices contain various generalizations and technical details.

2. Double-scaling the \((p, 1)\) models

Let us begin by considering a general form of the two-matrix model

\[
Z(g) = \int dA dB \ e^{-\frac{1}{g} Tr(V(A) + W(B) - AB)}
\]

where \(A\) and \(B\) are \(N \times N\) matrices, \(g\) is the coupling constant of the bulk theory, and the choice of integration contour depends on the form of the potentials \(V\) and \(W\). This model can describe \((p, q)\) minimal string theory in the large \(N\) limit [12], provided we tune the potentials \(V(A)\) and \(W(B)\). where we integrate \(z\) on a small contour around \(z = 0\), and \(X^*(z)\) and \(Y^*(z)\) have zeros of order \(q\) and \(p\), respectively, at \(z = 1\). We find it convenient to take

\[
X^*(Z) = \frac{(Z-1)^q}{Z} \quad \text{(2.2)}
\]

\[
Y^*(Z) = \frac{(Z-1)^p}{Z} \quad \text{(2.3)}
\]

In the large \(N\) double-scaling limit, this becomes the bulk partition function of the \((p, 1)\) model. Two comments on this result are in order: 1. For \(p = 2\), one can integrate out \(A\) to obtain a Gaussian model for \(B\). This case was studied in detail in [6]. Of course, \(A\) cannot be integrated out so easily in general. Nevertheless, the integral (2.6) is essentially trivial for any value of \(p\), since \(B\) always acts like a Lagrange multiplier constraining \(A\). This will facilitate much of the computation reported in this article. 2. Some care is necessary in order to ensure that the matrix integral is well defined. For even \(p\), this can be accomplished by integrating \(A\) and \(B\) with respect to the measure where \(\eta A\) and \(i\eta^{-1}B\) are Hermitian matrices, and \(\eta p = -1\). For odd \(p\), one can use the same measure, but one should integrate first over \(B\) so that the \(A\) integral is constrained.

3. Open/closed string correspondence

The main result of the previous section can be summarized as the statement regarding the expectation value of the exponentiated macroscopic loop operator in the double-scaling limit. These macroscopic loop operators, when parameterized by the length \(\ell\) of the boundary on the world sheet, can be decomposed into local closed-string operators in terms of their scaling [13-15] A similar relation between boundaries of fixed length and local operators (as well as a Laplace transformed version) also appears in section 30f [10]. One should keep in mind, however, that the expansion of macroscopic loop amplitudes in terms of microscopic operator correlation functions is subtle, in that there are divergent contributions as \(\ell = 0\) or \(y \rightarrow \infty\) in the disk and the annulus amplitudes [14]. These can be removed by introducing the normalization factor

\[
C(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{y^p y^p}{p} - \frac{p-1}{2p} \log y}
\]

which precisely cancels the divergences from the disk and the annulus amplitudes. One can then interpret the result of the previous section as the statement which relates the normalized FZZT brane partition...
function to the generating function of closed-string correlators in \((p, 1)\) topological string theory.

The relation is the simplest statement of open/closed string duality - it says that the partition function of the FZZT brane is equivalent to a certain shift in the closed-string background.

**Conclusion**

In this paper, we computed the exact partition function of (multiple) FZZT branes in the \((p, 1)\) topological background, using the two-matrix model in the double-scaling limit. We found that these partition functions are given by the generalized Kontsevich integral. By relating this to a specific insertion of closed string operators, we were able to formulate a precise open/closed string duality for \((p, 1)\) topological gravity with matter. In principle, the double-scaled matrix model can provide a framework for understanding open/closed string duality in the most general minimal string theory. However, one should keep in mind that our analysis was aided by various simplifications that occur in the \((p, 1)\) models. The most important simplification came in the calculation of the FZZT partition function at finite \(N\). In general, the FZZT partition function is the scaling limit of an orthogonal polynomial of the dual matrix model. For the \((p, 1)\) models, these orthogonal polynomials were sufficiently simple as to admit an elementary, closed-form representation. Furthermore, this representation allowed the scaling limit to be taken explicitly, which led directly to the generalized Airy function.

It would be interesting to push the matrix model analysis of open/closed string duality to more general \((p, q)\) models which are non-perturbatively well defined. Of course, the orthogonal polynomials for the general \((p, q)\) models are more complicated. Nonetheless, they are known to satisfy a recursion relation and can be generated in a finite number of steps. So there is still hope that one may be able to formulate concretely open/closed string duality for general \((p, q)\). Another open problem of immediate interest is to rederive the results in this paper using open-string field theory, as was done for \((2, 1)\) in [9]. (See also [25] for recent discussion on this issue.) This would presumably shed more light both on open/closed string duality in these models, and on the inner workings of open-string field theory itself. It would also confirm the identification of the generalized Kontsevich integral with the effective theory of open strings between FZZT branes. The primary motivation for studying explicit realizations of open/closed string duality in toy systems like \((p, 1)\) is to provide new insights that can be extended to richer dynamical systems. Let us briefly mention one possible lesson that could be learned from our work. So far, all attempts to formulate purely closed-string observables from open-string field theory in general [26,27] (see also the interesting recent work of [28]), following the work of [29-31], have had difficulty in removing the boundary of the world sheet. By contrast, here we encountered no such difficulties in formulating the open/closed string duality of the \((p, 1)\) model: purely closed-string amplitudes were easily obtained from the generalized Kontsevich matrix integral, and the latter presumably represents a reduction of open-string field theory along the lines of [9]. Nonetheless, there was one seemingly unnatural step where we removed the divergent contribution of the disk and the annulus amplitudes by hand. Finally, let us emphasize that, despite all the progress in making Witten’s conjecture mathematically precise [34-37], a proof of
(B.1) is still lacking for $p > 2$. That is, it remains to extend beyond $p = 2$ Kontsevich’s statement that the intersection numbers for $(p, 1)$ topological gravity are generated by the tau function of the $p$ reduced KP hierarchy. Such a generalization will presumably involve open string field theory techniques in order to generate the cell decomposition of the moduli space of spin curves. See [9,25] for a recent discussion on this issue.

References


